TIME: A NON-LINEAR MODEL (A SUMMARY: PART I)

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1. Introduction

In 1996 a paper was introduced by Carl L. DeVito titled *A Non-linear model of time* [1] (DNLT)[1], and presented during the International Conference on Modern Mathematical Models of Time and Their Applications to Physics and Cosmology, at the University of Arizona, in Tucson Arizona. This paper posed an entirely different perspective from the usual assumption that time is a *linear* quantity in the sense that it is a uni-directional "flow" in an ordered progression from past to future without any substantive reasons, other than our overwhelming intuitive sense, that there could be any alternatives. This linear model of time (LT) has framed our essential understanding of the creation and evolution of our physical universe and, in our modern perspective, it is further assumed that time may simply be considered an additional dimension of space within a *space-time* structure establishing the basis upon which the Special and General Theories of Relativity securely rest.[2] As well, within modern physics and cosmology both time and space have come under greater examination as demonstrated by recent controversies over the introduction of quantum universes (multiverses, many worlds, etc.) and alternate timelines. This has generated as many *proponents* (Stephen Hawking, Brian Greene, Alan Guth, Andrei Linde, Leonard Susskind and Sean Carroll), as *opponents* (Steven Weinberg, David Gross, Paul Steinhardt, Neil Turok, Michael S. Turner, Sir Roger Penrose, and George Ellis) and many more from each side. Albeit, to resolve a number of these controversies there has been much published on the physical and philosophical nature of space and time, it can be argued that time alone has suffered a lack of mathematical scrutiny for which DNLT is intended to remedy. Certainly Euclid, without any concept of time, introduced the geometry of space and established one of the most significant advances in human thought subjected to challenge and improvement only recently. As such, this article has a two-fold intent, *viz.* to introduce DNLT as a credible mathematical starting point and, hopefully, to generate more discussion on the physical and mathematical nature of time.[3]

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[1]This notation, DeVito Non-Linear Time is the author's and is not associated with DeVito's paper. Also, the term non-liner has several mathematical meanings and to alleviate any confusion we will use DNLT to be specific to DeVito's use of the term as related to his theory of time.

[2]This is not to say that this issue has not be considered, quite the contrary, *viz.*, McTaggart [2] in 1908 in *In The Unreality of Time* suggested that our perception of time is an "ideal" illusion and introduced two "series" A and B distinguishing temporal arrangements; and, Julian Barbour [3], in his 1999 book *The End of Time* has even suggested that time does not exist other than an "illusion" with the past as only a remnant of our memory and the future as simply our belief that it will occur.

[3]The author wishes to acknowledge that this summary is intended to familiarize the reader with DNLT as closely as possible to DeVito's original paper [1] and any misinterpretations or misrepresentations are strictly due to the author alone and if any, many apologies are extended to Professor DeVito. Also, any interpretations or additions by the author will be so noted as *"Remarks"* throughout this paper.

2. Preliminary Remarks

DeVito's intent is to answer two very fundamental questions, *viz.*, i) if the *standard linear model* (LT) has worked so well in several areas of science, why is there a need to replace it?; and; ii) if it is to be replaced, are there any advantages of succeeding it with something more complex? To answer these questions he constructs a *non-linear mathematical model of time* (DNLT) that incorporates a *linear component* (LT) as a necessity to which it must conform both intuitively and physically; and, most importantly, is *void of any spacial reference.* It is this latter condition that must be emphasized and within which these models differ significantly. Recalling that it is customary to treat time in the usual *space-time* model as simply a fourth "spacial" dimension thus having it share an assumed equal spacial weight, here, no such assumption is made.[4] It is within this new framework that DNLT lends itself an axiomatic structure similar to Euclid's treatment of space, *void of time*, thereupon placing it on mathematical *terra firma* from which future investigation may yield more intricate temporal models.

3. The Axioms and Definitions of DNLT

In the LT model it is intuitively clear that there is a natural temporal ordering between two *events*, A and B, as observed from one's inertial reference frame.[5] As such, our fundamental assumptions about temporal events, i.e. the past, the present, and the future have been framed within the strict limits to a *total ordering* analogous to the set of real numbers being represented as points on a real line [4]. However, in the context of the many aforementioned controversies surrounding the nature of time, the logical fallacy of a *false dichotomy* wherein it is either totally ordered or doesn't exist at all (see footnote 2) closes the door on any alternative temporal models.[6] As such, we abandon this restriction and introduce the possibility that time allows for a *partial ordering*, thus admiting now a series of axioms and definitions.

Axiom 1. We shall assume that time is an infinite set I whose elements will be called *instants*. Furthermore, we shall assume that the elements of I can be *partially ordered.*[7]

Notation. It is customary to use the familiar notation $\leq$ to represent the partial order [4] so if x and y are *instants* in I then "x $\leq$ y is to mean x is *no later than* y, and; when x $\leq$ y with x $\neq$ y to mean x *is earlier than y"* which we denote as x $<$ y [8][1] and, under these circumstances, x and y are said to be *comparable* in I. Let us recall that a partially ordering does admit the possibility that there exist elements x and y $\in I$ such that neither x $\leq$ y nor y $\leq$ x and in which case they will be denoted as *incomparable.* However, if there are comparable elements x, y, and z in I such that any two are comparable, say x $\leq$ y and y $\leq$ z then y is said to be *between* x and z, and similarly for z $\leq$ y and y $\leq$ x.

Axiom 2 generates the desired condition that between comparable elements x and y in I there is a reflection of a "length" or duration of time as in the usual LT model.

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[4]This is not completely accurate since space is bidirectional and time appears unidirectional identified by an *arrow of time* from past to future, e.g., a photograph of a sunrise might be confused with that of a sunset (reversing East and West), but viewing a video run in reverse never generates confusion, at least in our macroscopic universe. Thus far this distinction has not detracted us from treating LT in this manner.

[5]Although, by special relativity (SR), observers in different inertial frames may disagree *on the order of the events* as to which may occur first, second, or simultaneous, nonetheless there is an *observed* ordering with respect to that frame.

[6]DeVito makes no such claim but rather states "However, some events cannot be so ordered - perhaps they happen at the "same time" or they are so remote that we cannot tell which occurred first."[1] p. 358.

[7]For exactness all axioms will be stated verbatim from Devito[1] unless otherwise noted.

[8]It must be noted here that this is strictly taken verbatim from DeVito.

Axiom 2. There is a function denoted as dur that assigns to each pair of comparable instants x, y a non-negative, real number. Furthermore dur (x,y) = dur (y,x) for all comparable pairs x,y and dur (x,y) = 0 if, and only if, x = y (see footnote 7).

Remark(s). It must be remarked that although Axiom 2 initially appears to generate a LT, care must be taken here to distinguish this from DNLT. In LT there exists two unique points y and y' that are so many "minutes" say, from a point x, one in the past and the other in the future of x. However, in DNLT, there is no such uniqueness and there may exist many such instants that are so many minutes from x, thus obfuscating the concepts of past and future of a point x.

With respect to the last remark, Axiom 3 is almost immediate:

Axiom 3. Given any instant x and any positive, real number ρ we assume that there is at least one instant y such that dur(x,y) = ρ.

These initial axioms do suggest that in the set I there exist certain subsets DeVito calls "time tracks" that are *totally ordered* and, as such, all elements in a specific time track are comparable.

Remark(s). 1. Since any one time track adheres to all of the properties characteristic in the LT model, a time track is precisely the answer to the first question DeVito raises pertaining to its physical success. As he states, "In this view each object in the universe 'lives' on its own time track, and this track provides a kind of temporal 'base line' to which all external events may be referred."[1] p.360.

2. However, there is a fundamental difference between the LT and DNLT models, *viz.*, in the former there exists *one* time track for all observers; while in the latter, different observers have different time tracks. In this way DNLT opens up several alternative and less restrictive temporal theories, particularly if one observer is allowed to transit between time tracks (a point to which we will return in the sequel).

The preceeding remarks suggest our first definition:

Definition 1. A non-empty subset T of I will be called a **time track** if it has the three following properties:[9].

a) Any two instants on T can be compared;
b) If x, y, and z are on T and y is *between* x and z, then dur (x,z) = dur (x,y) + dur (y,z);
c) If Given y on T and a positive real number ρ there are exactly two instants x, z are on T such that dur (x,y) = ρ and dur (x,z) = ρ.

Remark(s). 1. It is evident that Definition 1, adhering explicitly to our "usual" concept of time, incorporates the LT model within DNTL, and;
2. In LT, I is itself a time track whereas in DNLT it is **not**, thus suggesting the next axiom.

Axiom 4. The set I is not a time track, but there is at least one time track in I.

4. Theorems and Further Definitions

Axioms 1 through 4 taken together with Definition 1, might mistakenly give the impression that a time track is coincident with the *physical time* as is customarily used in physics .[10][1] (p.360). We recall that in the LT model the assumption *ab initio* is made that time can be associated with

[9]As with the axioms, definitions will be stated verbatum from [1]

[10]Henceforth, the term physical time will be used in this context unless otherwise indicated

a real line axis as a spacial component in space-time. DNLT, quite the contrary, makes no such claim and because of this, care must be taken to avoid any such assumption. As a result, the following set of theorems and lemmata[11]are presented to establish the fact *terra firma* that a time track in DNLT maintains all properties that are normally assigned to *physical time* [12]

Theorem 1. *Let T be a time track, let y be an instant on T and ρ be a fixed, positive, real number. If x, z are the two instants on T such that dur(x,y) $= \rho$ and dur(y,z) $= \rho$, then y is between x and z.*

Definition 2. Let T be a time track and let ρ be a fixed real number. We define the translation function t_ρ, mapping T to T as follows: (a) t_0(x) = x for all x $\in T$; (b) If $\rho > 0$, t_ρ (x) = y where y is the unique instant on T such that x < y and dur(x,y) = ρ; (c) If $\rho < 0$, t_ρ (x) = z where z is the unique instant on T such that $z < x$ and dur(x,z) = $|\rho|$

Lemma 1. *For each fixed, real number ρ the function t_ρ, maps T onto T and is a one-to-one function.*

Theorem 2. *Let T be a time track and let ρ be a fixed real number. Then the function t_ρ from T to itself preserves order and duration. More explicitly, for any x, y on T we have: (a) $x \leq y$ if, and only if, $t_\rho(x) \leq t_\rho(y)$; (b) dur(x,y) = dur$[t_\rho(x), t_\rho(y)]$.*

Theorem 3. *If T_1, T_2 are time tracks, and if $T_1 \subseteq T_2$ then $T_1 = T_2$.*

Remark(s). 1. The preceding results seem at first to be self evident in preserving the three fundamental aspects of LT as used in physics, *viz.*, that time is (i) translational; (ii) a *real number*; and; (iii) maintains an "event" ordering. Caution is warranted here in the second instance, for although Axiom 2 ensures a *real number* temporal *duration* by virtue of the duration function, no such condition is imposed on the elements from the set of instants T. This does seems to suggest that there is no possibility of relating T to the usual numerical time axis as used in physics, however; this will be remedied below.

2. In the first and third instances, one is ensured by Theorems 1 and 2 to preserve *translational event ordering* and as DeVito states "The sequence of events breakfast-lunch-dinner can be thought of as happening on a given day. However, we can also imagine that they are translated through time by, say 24 hours." [1] (p.360);

3. It may also appear that these theorems are quite obvious, quite the contrary, the proofs are quite insightful and extensive and worthy of careful inspection. (See note 13).

4. Certainly, Theorem 3 is most interesting, proving that time tracks "are as large as possible."[1] (p. 362).

To conclude our Summary Part I, we address the concerns raised in Remark 1 regarding the transformation of any time track T into the usual "numerical time axis of mathematical physics." [1](p. 362). Firstly, by use of Definition 2 and Lemma 1, we construct a one-to-one function φ between the set of real numbers and any time track T in the following way: Let x $\in T$ and let $\varphi(0)$ = x, $\varphi(\rho) = t_\rho$(x) for all real ρ.[13] Also, we can define a function f from any arbitrary time track T into the reals to be *periodic* (noting that for physical time there are several periodic cycles of time, *viz.*, yearly, lunar, daily, etc.) such that if there exists some $\rho > 0$ such that f$[t_\rho$(x)] = f(x) $\forall$ x $\in T$. Thus having established this one-to-one correspondence between T and the reals and

[11]All theorems and lemmata are written verbatum from DeVito.

[12]All theorems and lemmata are stated without proof, however, for a greater understanding of the material it is highly advised to read through the proofs in the original article. Note: The author will, upon request, supply proofs to those interested: apitucco@comcast.net.

[13]It should be noted as well that by this correspondence we are ensuring that any point x in T can be assigned an "initial" 0 time from the usual time axis in physics.

recognizing that the time track of the Earth may include the usual physical periodicities makes certain that the physical LT model is imbedded within DNLT.

Finally, although this summary posits a set of definitions, axioms, and theorems upon which additional mathematical structure may be placed, the author also wishes to create an avenue for further discussion of DeVito's model of Non-linear Time. It is with this intent that we conclude our Part I summary in order for readers to have the opportunity to discuss and generate questions on these initial formulations.

To be continued in the sequel, *viz.,* Part II .

References

[1] Carl L. DeVito *A Non-linear model for time*. In W.G. Tifft and W.J. Cocke (Eds.) *Modern Mathematical Models of Time and their Applications to Physics and Cosmology*: Kluwer Academic Publishers, Dordrecht. Reprinted from *Astrophysics and Space Science*, Vol. 244: 357-369, 1996 Nos. 1-2, (1996).

[2] John McTaggart Ellis McTaggart *The Unreality of Time* 1915:The Perfect Library.

[3] Julian Barbour *The End of Time: The Next Revolution in Physics* 2001: Oxford University Press: Reprint edition.

[4] Pitucco, Anthony. "Binary Relations And Partially Ordered Sets". The Anthony Pitucco Blog 2016. Web. 13 Jan. 2016. https://anthonypitucco.wordpress.com/2016/01/12/binary-relations-and-partially-ordered-sets/

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